Do CDOs Work?

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Do CDOs Work?

Background

Economics of a CDO

Empirical test and examples

Conclusions
• In the period 1998 to 2007, CDOs increased exponentially in both volume and diversity
  – Prior to 2007, the CDO was seen as a successful financial innovation

• However, the global financial crisis was partly catalysed by an implosion in the CDO market and caused massive losses for:
  – Issuers (banks) through investments held, litigation, failed hedges, reputation
  – Investors, both in terms of default losses and those from forced liquidation
  – Third parties (e.g. rating agencies through loss of fees, reputation issues and litigation)

• An obvious question is therefore:
  – Is there something fundamentally wrong with the concept of a CDO?
  – Does it have economic value or is just a money making tool for investment bankers?
Assumptions

- This analysis will be based on a CDO under the following assumptions
  - Full capital structure (although this is not especially important)
  - Static portfolio (again particularly important as we care mainly about the initial portfolio)
  - Corporate credit risk (due to the richer data than for asset backed structures)
  - The ratings process used by ratings agencies for CDO structures during the period in question

- A CDO is broadly speaking
  - An investment at risk to a pre-defined range of losses on a certain portfolio
  - As such, the risk assessment requires an analysis of the multidimensional default distribution (which is quite complex)
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## Example CDO

<table>
<thead>
<tr>
<th>Class</th>
<th>Amount</th>
<th>Tranching</th>
<th>Rating</th>
<th>Funding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Super senior</td>
<td>850</td>
<td>[15-100%]</td>
<td>NR</td>
<td>Unfunded</td>
</tr>
<tr>
<td>Class A</td>
<td>50</td>
<td>[10-15%]</td>
<td>Aaa/AAA</td>
<td>Funded</td>
</tr>
<tr>
<td>Class B</td>
<td>30</td>
<td>[7-10%]</td>
<td>Aa2/AA</td>
<td>Funded</td>
</tr>
<tr>
<td>Class C</td>
<td>30</td>
<td>[4-7%]</td>
<td>Baa2/BBB</td>
<td>Funded</td>
</tr>
<tr>
<td>Equity</td>
<td>40</td>
<td>[0-4%]</td>
<td>NR</td>
<td>Funded</td>
</tr>
</tbody>
</table>
Example CDO Economics

- Very simple example (more rigorous one later)

<table>
<thead>
<tr>
<th>Tranching</th>
<th>Rating</th>
<th>Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baa [0-100%]</td>
<td>Baa</td>
<td>250</td>
</tr>
<tr>
<td>Aaa [8-100%]</td>
<td>Aaa</td>
<td>100</td>
</tr>
<tr>
<td>Ba [4-8%]</td>
<td>Ba</td>
<td>600</td>
</tr>
<tr>
<td>Caa [0-4%]</td>
<td>Caa</td>
<td>1500</td>
</tr>
</tbody>
</table>

**Excess Spread**

\[
= (250 - 100 \times 92\% - 600 \times 4\% - 1500 \times 4\%)
\]

\[= 74 \text{ bps}\]

Due to the short duration of the equity tranche, the excess spread is a conservative measure of the profitability of a CDO.

If this number is large and positive then things are looking good.

Triple-B portfolio
• Suppose there is a continuum of underlying tranches (full capital structure)
  – Each tranche is denoted by $i$
  – The underlying portfolio is denoted by $p$

• Consider expected loss as the main quantitative characteristic of the tranche
  – Expected loss must be conserved across the structure

$$EL_P = \sum_i m_i EL_i$$

where

- $EL_P$ is the expected loss for the entire portfolio
- $m_i$ is the tranche size
- $EL_i$ is the expected loss for unit tranche (under physical measure)

Also, $\sum_i m_i = 1$ for the entire portfolio.
Economics of a CDO (2)

• Investors will demand a premium for the losses they take
  – Let us represent this as a multiplier $\alpha$ which varies for the different tranches and original portfolio and therefore represents the risk aversion for a particular seniority
  – Investors will be paid $\alpha_i m_i E L_i$
  – The CDO will “work” if
    $$\alpha_p E L_p > \sum_i \alpha_i m_i E L_i$$
  – This basically requires that it is possible to buy protection cheaper via the CDO tranches than it is on the underlying portfolio

• Note that the $\alpha$ will be determined via the coupon demanded on the various tranches by investors
Risk aversion by seniority

- **How do we represent** $\alpha$?
  - The primary consideration of investors was the rating of the underlying tranche
  - In turn, the fundamental driver of ratings would be the expected loss of a tranche (or default probability in the case of Standard & Poor’s)
  - Hence we assume

$$
\alpha_j = \left( \frac{a}{EL_j} \right)^b
$$

- **Properties**
  - Risk-neutral investors, $b = 0$
  - Risk aversion for $a, b > 0$
  - More relative risk aversion for small expected losses
Criteria for a CDO to work

- **What parameters are required for a CDO to work?**
  - We require:
    \[
    \alpha_p E L_P > \sum_i \alpha_i m_i E L_i
    \]
    \[
    \alpha_j = \left( \frac{a}{E L_j} \right)^b
    \]
  - Which becomes:
    \[
    \left( \frac{a}{E L_P} \right)^b E L_P > \sum_i \left( \frac{a}{E L_i} \right)^b m_i E L_i
    \]
  - Simplifying to:
    \[
    E L_P^{1-b} > \sum_i m_i E L_i^{1-b}
    \]
  - Which is satisfied when \( b < 1 \)
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- **Hull, Predescu and White (2005)**
  - Time period, December 1996 to July 2004
  - Merrill Lynch bond indices and Moody’s data

<table>
<thead>
<tr>
<th></th>
<th>Default intensity</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real world</td>
<td>Risk-neutral</td>
<td>Ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aaa</td>
<td>4</td>
<td>67</td>
<td>16.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aa</td>
<td>6</td>
<td>78</td>
<td>13.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>13</td>
<td>128</td>
<td>9.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baa</td>
<td>47</td>
<td>138</td>
<td>5.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ba</td>
<td>240</td>
<td>507</td>
<td>2.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>749</td>
<td>902</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caa</td>
<td>1690</td>
<td>2130</td>
<td>1.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assume recovery rate of 40%

\[ a = 0.24, \ b = 0.47 \]
Back to a simple example

- **Rating assumptions**
  - Expected loss based
  - Gaussian copula approach with flat correlation of 20%

<table>
<thead>
<tr>
<th>Rating</th>
<th>Tranche</th>
<th>5-year exp loss</th>
<th>Multiplier</th>
<th>Protection value</th>
<th>Size</th>
<th>Spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baa</td>
<td>[0-100%]</td>
<td>1.296%</td>
<td>5.1</td>
<td>6.610%</td>
<td>100%</td>
<td>144</td>
</tr>
<tr>
<td>Aaa</td>
<td>[8-100%]</td>
<td>0.072%</td>
<td>16.8</td>
<td>1.210%</td>
<td>92%</td>
<td>26</td>
</tr>
<tr>
<td>Ba</td>
<td>[4-8%]</td>
<td>6.702%</td>
<td>2.1</td>
<td>14.074%</td>
<td>4%</td>
<td>321</td>
</tr>
<tr>
<td>Caa</td>
<td>[0-4%]</td>
<td>36.498%</td>
<td>1.3</td>
<td>47.447%</td>
<td>4%</td>
<td>1376</td>
</tr>
</tbody>
</table>

Excess Spread
= (144
−26 × 92%
−321 × 4%
−1376 × 4%)
= 52 bps

Net protection value
= 6.610% − 1.210% × 92% + 14.074% × 4% + 47.447% × 4% = 3.036%


• Assuming investors demand a return based on the expected loss (via the rating) of a tranche
  – A CDO always “works” due to the risk preferences of investors (the equity tranche is relatively cheap to get rid of due to the small alpha multiplier)

• Another implication of this is that rating agency modelling assumptions cannot cause the CDO to fail
  – For example, let us look at correlation assumptions
Excess spread as a function of correlation

- Excess spread as a function of flat correlation assumptions in rating model
  - CDO clearly “fails” at high correlation
• Previous failure was due to the granularity in the ratings process

• Therefore we assume a simple optimisation
  – Make the equity tranche small enough to just support a given rating (Caa is best)
  – Find the size of the mezzanine tranche to give the best excess spread
• Now the CDO works at all correlation levels

• Note there is still some inherent granularity
  – Can’t get any worse than Caa or better than Aaa
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• **A CDO works due to**
  – The risk preferences of investors
  – The expected loss methodology used in the ratings process

• **A CDO is not a zero sum game**
  – Both issuers and investors (and third parties) can gain
  – Just because an issuer makes money, no direct implication that investors are getting a bad deal

• **Rating agencies were not at fault?**
  – No modelling assumptions would have caused CDOs to be unprofitable
  – Although rating agencies primary reliance on quantitative models based on expected loss as the only metric could be seen as too simplistic and a fundamental flaw
So what did go wrong? (with CDOs at least)

• Lack of proper assessment of counterparty risk in the structuring process
  – The more senior the tranche, the more counterparty risk (relatively) – see my book!
  – Large senior tranches were offloaded to monoline insurers without any collateral terms to mitigate the counterparty risk
  – E.g. see Gregory, “A free lunch and the credit crunch”, Risk, August 2008

• Lack of appreciation of the systemic risk in senior tranches
  – Were investors sufficiently compensated for this?

• Maybe there is sufficient value in a CDO to overcome the above problems
  – But the market was too greedy and now it may be too late!