Gaining From Your Own Default
– The Strange Case of DVA

Jon Gregory

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Outline

1. Background, accounting rules and examples
2. CVA (credit value adjustment)
3. CVA and capital
4. DVA (debt value adjustment)
5. How to realise DVA
6. DVA and funding
Counterparty Casino:
The need to address a systemic risk

By Jon Gregory
Background, Accounting Rules and Examples
The Trials of Regulation (I)

• What don’t I like as a regulator?

• Different institutions valuing assets differently
  – Institution A trades a derivative with institution B and they both book a profit!

• Institutions making profits based on “mark-to-model”
  – By the time we realize our model was wrong then bonuses have been paid……

• Balance sheets not being a zero sum game
  – For example, if a firm issues a bond do they mark its par value as a liability or its market value?
The Trials of Regulation (II)

• How to solve the problems?

• Different institutions valuing assets differently
  – Mark-to-market (fair value accounting)

• Institutions making profits based on “mark-to-model”
  – Mark-to-market

• Balance sheets not being a zero sum game
  – Mark-to-market (of own liabilities on balance sheet)
Pricing Liabilities With Your Own Credit Risk

- Suppose a firm issues a bond (par value $100) with a treasury like coupon
- The market will only pay $95 for this bond due to the firm’s credit risk
Gaining from Your Own Default

• The firm’s credit spread widens
• The market price of the bond is now $90
• Profit of $5

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>.............</td>
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<td>.............</td>
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<tr>
<td>$95 cash</td>
<td>$90 bond</td>
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</tbody>
</table>

18% of pre-tax income for JPM, MS, BoA and GS in second quarter
CVA
History of Counterparty Risk and CVA

CCR / CVA Timeline
In a few short years we have seen a paradigm shift in CCR with the transition from Passive to Active management of CVA that requires ever more accurate and more frequent CVA calculations – daily, intra-daily, and real-time.

Before CVA
- Firms apply credit limits and measures such as PFE (Potential Future Exposure) to limit their possible exposure to a counterparty in the future

1999 Passive Management of CVA
- Large banks first start using CVA to assess the cost of counterparty risk
- CVA is treated via a passive insurance style approach

2007 Active Management of CVA
- The Credit Crisis and resulting failures of high profile firms generates much more attention on counterparty risk
- Banks are interested in more accurate and ever more frequent CVA calculations – daily, intra-daily, and real-time

1998: Asian crisis and long-term capital management (LTCM). The unexpected failure of the large hedge fund LTCM and asian crisis lead to an interest in CCR although mainly confined to some first tier banks

2006: New Accountancy regulations (FASB 157, IAS 39) mean that the value of derivatives positions must be corrected for counterparty risk
- All banks must start calculating CVA on a monthly basis

Sept. 10-15, 2008: Lehman Brothers collapses following a reported $4 billion loss and unsuccessful negotiation to find a buyer, one of Wall Street’s most prestigious firms files for bankruptcy protection

Source: Algorithmics
CVA (Credit Value Adjustment)

• CVA is the price of counterparty risk (expected loss) and is a **cost**

Risk**y Derivative = Derivative - CVA

• Crucial to be able to separate valuation of derivatives and their CVA
  (below formula assumes no wrong way risk)

\[
CVA(t) = (1 - \delta_C) \int_t^T EE(u) dPD_C(u)
\]

- **Percentage recovery value**
- **Expected exposure including discounting** (how much we expect to lose)
- **Default probability** (how likely is counterparty to default at this time)
But CVA is Very Complex

• CVA represents an option on an underlying derivative
  – CVA calculation always harder than pricing the derivative itself

• Need the default probability (and recovery rate) of the counterparty
  – Often market implied probabilities are not known (no CDS market)

• Derivatives are subject to netting agreements
  – Need to price all other trades with this counterparty as well as trade in question
  – All correlations (same asset class, cross-asset class must be known)

• Wrong way risk
  – Linkage between default probability and exposure at default

• Collateral agreements, break clauses etc
CVA – Risk-Neutral or Not?

• Actuarial
  – Consistent with loan book management
  – Insurance company style approach is easier
  – No hedging

• Risk-neutral
  – Consistent with derivatives valuation
  – But trading function for CVA is very difficult to run
  – Hedging is extremely difficult or impossible

• Regulators favour the risk-neutral (mark-to-market) approach
  – But recent problems with hedging in the turbulent Eurozone possibly question this
  – And loans are not treated this way (a derivative is essentially an exotic loan)
CVA and Capital
Alpha and Basel II

![Graph showing Loss Probability and Unexpected Loss Approximation vs. Actual for EE Approx and Actual models. The graph illustrates the distribution of loss probabilities across different loss percentages.]
Alpha as defined in Basel II

- Basel 2 requires capital to be held against derivatives exposures
- Calculation covers
  - Default risk
  - Credit migration risk (through maturity adjustment factor)
- Alpha adjusts for
  - Exposure volatility
  - Correlation of exposures
  - Size of portfolio (and granularity)

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>Infinitely large portfolio and independent exposures (theoretical result only)</td>
</tr>
<tr>
<td>1.4</td>
<td>Supervisory value</td>
</tr>
<tr>
<td>1.2</td>
<td>Supervisory floor when bank uses own model for estimate</td>
</tr>
<tr>
<td>1.05 - 1.10</td>
<td>Typical value for large portfolios</td>
</tr>
<tr>
<td>&gt; 2.5</td>
<td>Possible value for concentrated portfolios</td>
</tr>
</tbody>
</table>
Regulatory Reaction to the Credit Crisis

- BCBS Committee (Dec 2009)
  - …. where current treatment did not adequately capitalise for risks during the crisis

- Key problems identified
  - Capitalisation of CVA volatility (2/3 of counterparty risk related losses during crisis?)
  - Initial margining (capital to give incentive for adequate initial margin through cycle)
  - Central counterparties not utilised
  - Close-out periods
  - Interconnection of financial institutions
  - Lack of back-testing and stress testing
  - Wrong-way risk
Basel 3 Proposal – CVA “VAR”

- Previous Basel 2 rules account only for default losses (and to some extent credit migration losses)

- Simple capital add-on for CVA risk (bond equivalent)
  - Notional of bond is defined by quantifying future exposure
  - Spread is the one used to calculate CVA (actual or proxy)
  - Maturity of bond is maximum effective maturity of all netting sets for that counterparty

- Risk is then defined as a market risk charge on this bond portfolio
  - VAR type 99% confidence level and 1-year period (may use scaled 10-day)
  - Accounts for hedging using single name CDS and CCDS (or similar instruments) only
The Problems With CVA VAR

• Recent changes
  – Remove the multiplier of 5 (scaling from 10 days to 1 year) 😊

• Only single name hedges (CDS, CCDS) given capital relief
  – Now seemingly will give some relief for index hedges
  – But how? And will this not be encourage procyclicality?

• Methodology
  – Intended to capture in a simple way the credit spread risk within CVA
  – Actually, it is not the optimal way to do this and can lead to non economic results (Rebonato et al.)

• Motivation
  – OTC derivatives are relatively precisely valued, their VAR is much harder to quantify
  – CVA itself is hard to quantify so CVA VAR is surely a major challenge?
DVA
Unilateral CVA in the Old Days

<table>
<thead>
<tr>
<th>Credit Rating</th>
<th>Credit spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank</td>
<td>Aa1/AA+</td>
</tr>
<tr>
<td>Corporate</td>
<td>A3/A-</td>
</tr>
</tbody>
</table>

• Bank has no default risk
  – Bank charges corporate unilateral CVA
  – If corporate asks for banks default probability to be taken into account, they get laughed at
• No CVA charges in interbank market (collateralised, banks won’t default)
• When bank credit quality deteriorates, market becomes gridlocked
Pricing Bilateral Counterparty Risk

- Bilateral CVA considers also an institution's own default (this formula assumes independent of defaults)

\[ BCVA(t) = (1 - \delta_C) \int_T^t EE(u)[1 - PD_I(u)]dPD_C(u) \]

\[ -(1 - \delta_I) \int_T^t NEE(u)[1 - PD_C(u)]dPD_I(u) \]

CVA

DVA

- Own percentage recovery value

- Negative expected exposure

- Probability counterparty hasn't yet defaulted

- Probability we default

- Expected exposure

- Probability we haven't yet defaulted

- Probability counterparty defaults
Computing the Bilateral Price

- **Bilateral CVA Example**
  - Case A: Counterparty 250 bps CDS, Institution 500 bps CDS, EE < NEE
  - Case B: Counterparty 500 bps CDS, Institution 250 bps CDS, EE > NEE

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
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</thead>
<tbody>
<tr>
<td>CVA</td>
<td>1.235%</td>
<td>3.480%</td>
</tr>
<tr>
<td>BCVA</td>
<td>-1.967%</td>
<td>1.967%</td>
</tr>
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</table>
Default Correlation

• Gaussian copula approach can be used to give simple tractable correlation between our own default and that of our counterparty
  – Just requires bivariate Gaussian distribution function
  – For example, probability our counterparty defaults in an interval but we don’t

\[
Q(\tau_C \in [t_{i-1}, t_i], \tau_I > t_i, \tau > t_i) = Q(\tau_C > t_{i-1}, \tau_I > t_i, \tau > t_i) - Q(\tau_C > t_i, \tau_I > t_i, \tau > t_i)
\]

\[
\approx \left[ \Phi_{2d} \left( \Phi^{-1} \left( Q(\tau_C > t_{i-1}) \right), \Phi^{-1} \left( Q(\tau_I > t_i) \right); \rho \right) \right] Q(\tau_C > t_i)
\]

\[
- \left[ \Phi_{2d} \left( \Phi^{-1} \left( Q(\tau_C > t_i) \right), \Phi^{-1} \left( Q(\tau_I > t_i) \right); \rho \right) \right] Q(\tau_I > t_i)
\]
Impact of Correlation on BCVA

- Case B from previous example
  - Counterparty 500 bps CDS, Institution 250 bps CDS, EE > NEE

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**Base Case**

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Impact of DVA

Bilateral CVA $\approx EPE \times \text{Counterparty spread} - ENE \times \text{Institution spread}$

Net adjustment to derivatives book
Does Bilateral CVA Make Sense?

- Bilateral CVA has been widely adopted
  - Many banks base CVA on their own default
  - Accountancy rules require this (e.g. FAS 157)

- Bilateral CVA has some potentially unpleasant features
  - Total amount of CVA in the market sums to zero
  - Risky value may exceed risk-free value
  - Netting and collateral may increase CVA+DVA
  - Hedging this component is problematic

- How to monetise bilateral CVA to justify paying for counterparty risk?
How to Realise DVA

• Go bankrupt
  – Usually not a popular choice

• Unwinds or novations
  – An institution may realise a DVA gain if a trade is unwound in the future (e.g. banks unwinding transactions with monolines)

• Hedging
  – DVA much harder to hedge than CVA - cannot sell CDS protection on yourself!
  – Buy back your own debt (not really a dynamic hedge) – do you have the cash?
  – Sell CDS on another counterparty (who is highly correlated with you) – give wrong-way risk to buyer of protection – careful who you choose (Lehman)

• Funding arguments
  – EE represents a funding cost, NEE represents a funding benefit
Hedging Intuition of DVA

- Following Sorenson and Bollier [1994]

\[
CVA_{\text{swap}} \approx (1 - \delta_C) \sum_{i=1}^{n} S_I(t_{k-1})[S_C(t_{k-1}) - S_C(t_k)]V_{\text{swaption}}(t; t_k, T)
\]

\[
DVA_{\text{swap}} \approx (1 - \delta_I) \sum_{i=1}^{n} S_C(t_{k-1})[S_I(t_{k-1}) - S_I(t_k)]V_{\text{swaption}}(t; t_k, T)
\]

- Intuition
  - Short a series of swaptions (on reverse swap) with weights given by the forward default probabilities (of counterparty)
  - Long a series of swaptions (on reverse swap) with weights given by the forward default probabilities (of self)

- Hence, using DVA may balance sensitivities
**Hedging Using DVA (I)**

- **Sensitivity to interest rates**
  - If CVA increases (for example interest rates go up for a payer swap)
  - Then DVA will decrease
  - Overall sensitivity is increased

![Diagram showing sensitivity to interest rates for unilateral and bilateral DVA over different tenors (1Y, 2Y, 3Y, 4Y, 5Y). The diagram uses bars to represent the sensitivity values, with blue bars for unilateral and red bars for bilateral DVA.]
Hedging Using DVA (II)

- Sensitivity to volatility
  - Long and short swaptions will cancel
  - In this case we are half as risky as counterparty (CDS = 250 bps vs 500 bps)
  - Sensitivity is approximately halved

![Diagram showing CVA sensitivity for different swap rates and volatility levels. The bars indicate unilateral and bilateral CVA sensitivity for 1Y, 2Y, 3Y, 4Y, and 5Y swap rates. The y-axis represents CVA sensitivity ranging from 0.00% to 0.35%, and the x-axis represents swap rate volatility with years labeled 1Y, 2Y, 3Y, 4Y, and 5Y.]
Hedging Using DVA (III)

- Impact of DVA on CDS hedges
  - Buy slightly less protection on counterparty (due to possibility of self defaulting first)
  - Sell protection on oneself
  - Actually made easier by the absence of single name hedges (index beta effect)
DVA and Funding
# Funding Costs and CVA / DVA

<table>
<thead>
<tr>
<th>Measure</th>
<th>Exposure</th>
<th>Default probability</th>
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<tbody>
<tr>
<td><strong>Default</strong></td>
<td></td>
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</tr>
<tr>
<td>Default</td>
<td>CVA</td>
<td>EPE</td>
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<tr>
<td></td>
<td>DVA</td>
<td>ENE</td>
</tr>
<tr>
<td><strong>Funding</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Funding cost</td>
<td>Funding cost</td>
<td>EPE</td>
</tr>
<tr>
<td>Funding benefit</td>
<td>Funding benefit</td>
<td>ENE</td>
</tr>
</tbody>
</table>

*Double counting*
Double Counting of Funding

- **CVA of a single cashflow**

\[
CVA = \mathbb{E}\left[ e^{-(r+X_I)T} \tau_C > T \right] = e^{-rT} \times e^{-X_I T} \times e^{-X_C T}
\]

\(X_I = \) Funding spread

- **DVA**

\[
DVA = \mathbb{E}\left[ e^{-(r+X_I)T} \tau_I > T \right] = e^{-rT} \times e^{-X_I T} \times e^{-X_I T} = e^{-rT} \times e^{-2X_I T}
\]

\(X_I = \) Funding gain

\(X_I\) (own)
Funding and DVA – Some Relevant Papers

• Fries, C., 2010, “Discounting revisited: valuation under funding, counterparty risk and collateralization”

• Morini and Prampolini., 2010, “Risky funding: a unified framework for counterparty and liquidity risk”

• Piterbarg, V., 2010, “Funding beyond discounting: collateral agreements and derivatives pricing”

• We’ll follow the Morini and Prampolini notation but ignore the CDS-bond basis and assume zero recovery rates

• Note we are considering the case of no CSA (collateral)
Funding and DVA – The Key Concept

• For unsecured funding, I pay a funding spread of say $X_I$
• But I don’t pay the funding back if I default $\tau_I$
• Hence, when I pay back the funding of $L$ a time $\Delta t$ later, I pay

$$Le^{r\Delta t} e^{X_I \Delta t} 1_{\tau_I > T}$$

• The discounted expectation of this is then

$$Le^{-r\Delta t} e^{r\Delta t} e^{X_I \Delta t} e^{-X_I \Delta t} = L$$

• Funding cost therefore doesn’t depend on my credit spread
• This is the accountants view but should it be the quants view?
The Simple Derivative

- We can think of this as a simple swap with only 2 possible market scenarios and one time period.

Binary event

$B$

Exposure of $L$
- Owed money
- Funding cost

$\Delta t$

Rec from counterparty $Le^{r\Delta t}$
Pay back funding

$1 - B$

Exposure of $-L$
- Owe money
- Funding benefit

$\Delta t$

Pay to counterparty $Le^{r\Delta t}$
Release funding
Case 1

- This is similar to a contingent swap or clean asset swap (swap cancelled on the basis of a credit event A) with risk-free counterparties.
Contingent Swap – Valuation

Payoff = \[ B \left[ -L e^{(r+X_I)\Delta t} + L e^{r\Delta t} \right] 1_{\tau_A > \Delta t} + (1 - B) \left[ L e^{(r+X_I)\Delta t} - L e^{r\Delta t} \right] 1_{\tau_A > \Delta t} \]

- Price (no wrong way risk)

\[ V = E[B] \left[ -L e^{X_I\Delta t} + L \right] e^{-X_A\Delta t} + E[1 - B] \left[ +L e^{X_I\Delta t} - L \right] e^{-X_A\Delta t} \]

- Funding cost
- Rec
- Funding benefit
- Pay

- If \( E[B] = q = 1 - q \) then \( V = 0 \) with hedging implication that we need to hedge market risk and buy or sell protection on credit A and consider the need to charge for these dynamic hedging costs

- Mirror trades then Payoff = 0
Case 2

This is similar to a risky swap with counterparty risk where we consider ourselves default free (by the market does not of course)
Unilateral Risky Swap – Valuation

Payoff = \[ B \left[ -L e^{(r+X_I)\Delta t} + L e^{r\Delta t} \mathbb{1}_{\tau_A > \Delta t} \right] + (1-B) \left[ + L e^{(r+X_I)\Delta t} - L e^{r\Delta t} \right] \]

\[
V = E[B] \left[ -L e^{-X_I\Delta t} + L e^{-X_A\Delta t} \right] + E[1-B] \left[ + L e^{-X_I\Delta t} - L \right]
\]

- Mirror trades with two different counterparties A and B
  \[
  V_{AB} = E[B] \left[ -L e^{-X_I\Delta t} + L e^{-X_A\Delta t} + L e^{-X_I\Delta t} - L \right] + E[1-B] \left[ + L e^{-X_I\Delta t} - L - L e^{-X_I\Delta t} + L e^{-X_B\Delta t} \right]
  \]
- Funding cancels, the trade has negative value for \( X_A, X_B > 0 \)
  \[
  V_{AB} = E[B].L \left[ e^{-X_A\Delta t} - 1 \right] + E[1-B]L \left[ e^{-X_B\Delta t} - 1 \right]
  \]
Case 3

- This is similar to a risky swap where both counterparties may default
Bilateral Risky Swap – Valuation (I)

Payoff = \( B \left[ -Le^{(r+X_A)\Delta_t}1_{\tau_I > \Delta_t} + Le^{r\Delta_t}1_{\tau_A > \Delta_t} \right] + (1 - B) \left[ Le^{(r+X_I)\Delta_t}1_{\tau_I > \Delta_t} - Le^{r\Delta_t}1_{\tau_I > \Delta_t} \right] \)

\[
V = E[B] \left[ -L + Le^{-X_A\Delta_t} \right] + E[1 - B] \left[ +L - Le^{-X_I\Delta_t} \right]
\]

- If \( E[B] = q = 1 - q \) \( V = Le^{-r\Delta_t} \left[ e^{-X_A\Delta_t} - e^{-X_I\Delta_t} \right] / 2 \)
- Funding cancels in expectation (but still have funding risk)
- Hedging implications
  - Hedge market risk
  - Buy protection on A, sell protection on ourselves
  - Consider hedging costs even when \( X_A = X_I \)
Bilateral Risky Swap – Valuation (II)

• Mirror trades with two different counterparties A and B

\[
\text{Payoff} = B \left[ -Le^{(r+X_I)\Delta t} 1_{\tau_I > \Delta t} + Le^{r\Delta t} 1_{\tau_A > \Delta t} + Le^{(r+X_I)\Delta t} 1_{\tau_I > \Delta t} - Le^{r\Delta t} 1_{\tau_B > \Delta t} \right] \\
+ (1 - B) \left[ Le^{(r+X_I)\Delta t} 1_{\tau_I > \Delta t} - Le^{r\Delta t} 1_{\tau_I > \Delta t} - Le^{(r+X_I)\Delta t} 1_{\tau_I > \Delta t} + Le^{r\Delta t} 1_{\tau_B > \Delta t} \right]
\]

• Funding cancels

\[
\text{Payoff} = Be^{r\Delta t} \left[ + L1_{\tau_A > \Delta t} - L1_{\tau_I > \Delta t} \right] + (1 - B)e^{r\Delta t} \left[ - L1_{\tau_I > \Delta t} + L1_{\tau_B > \Delta t} \right]
\]

• Valuation

\[
V_{AB} = E[B]L \left( e^{-X_A\Delta t} - e^{-X_I\Delta t} \right) + E[1 - B]L \left( e^{-X_B\Delta t} - e^{-X_I\Delta t} \right)
\]

• Same comments as before on hedging

• In this case **DVA is clearly not a funding benefit**
Should you use DVA?

• On the one hand, firms need to use DVA
  – Reduces CVA charges
  – Likely that both counterparties to a trade will agree a price
  – Reduces volatility of CVA desk’s book and hedging costs

• On the other hand
  – **Cannot be treated as a funding benefit**
  – Requires a firm to see their future default as a good thing and try and monetise it
  – Does not encourage good practices for a CVA desk
  – For example, a firm going to default will need to sell more and more CDS protection (and more and more volatility)